Your Signature ______________________

This is a closed book exam. Show all your work. Correct answers with insufficient or incorrect work will not get any credit. Maximum possible score is 100. There are six questions.

Score

1.	(10)	
2.	(15)	
3.	(15)	
4.	(20)	
5.	(25)	
6.	(15)	
Total.	(100)	

Extra sheets attached(if any): ______________________

Your Signature ______________________

1. Assume that a particular type of radio active material is in an environment that disintegrates at a rate, α, proportional to the material present. Find the time required for the mass to reduce to one-half its size.

2. Using the method of characteristics find the solution $u : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{aligned} x\frac{\partial u}{\partial y} - y\frac{\partial u}{\partial x} &= u \text{ on}\{x > 0, y > 0\} \\ u(x,y) &= g(x) \text{ on } L\{x > 0, y = 0\} \end{aligned}$$

3. Find the two linearly independent Frobenius series solutions to the ordinary differential equation

$$2t\frac{d^2x}{dt^2}(t) + (3-t)\frac{dx}{dt}(t) - x(t) = 0,\ t > 0.$$

4. Consider the ordinary differential equation

$$\frac{d^2x}{dt^2}(t) + 4\frac{dx}{dt}(t) + 4x(t) = t^{-2}e^{-2t},\ t > 1.$$

   (a) Find the general solution.

   (b) Find the solution $x(\cdot)$ such that $x(1) = 0$, $\frac{dx}{dt}(1) = -e^{-2}$.

5. (a) If $p(r,\alpha,\theta)$, $r < 1, -\pi \le \phi < \pi, -\pi \le \theta < \pi$, is the Poisson Kernel in the unit disc then show that

$$\int_{-\pi}^{\pi} p(r,\alpha,\theta)d\theta = 1.$$

   (b) Find the solution to the Dirichlet Problem

$$\begin{aligned} \Delta u(r,\theta) &= 0\ 1 < r < 2, -\pi \le \theta < \pi \\ u(1,\theta) &= 1 + \cos^2(\theta) - \pi \le \theta < \pi \\ u(2,\theta) &= 1 - \cos^2(\theta) - \pi \le \theta < \pi \end{aligned}$$

   Verify that the solution is indeed harmonic in the annulus.

6. Find the solution $u : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$, which is bounded in x and satisfies

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} + 3t^2,\ t > 0, x \in \mathbb{R}$$

   with $u(0,x) = \sin(x), x \in \mathbb{R}$.